Magnetic Field

- Magnetic effects occur at a distance without need for physical contact: existence of magnetic field

- Magnetic field arises around:
  - magnetized material
  - any *moving* electric charge (electrical currents)

- Historically, symbol $B$ is used to represent magnetic field

- Magnetic field structure is represented through magnetic field lines
Magnetic Field

- Poles of magnet have similarities to electric charges:
  - magnetic poles exert attractive or repulsive forces on each other: opposites attract, likes repeal
  - these forces vary as inverse square of distance between interacting poles

- Major differences between electric charges and magnetic poles:
  - Electric charges „+” and „−” can be isolated (e.g., electron and proton)
  - Single magnetic pole has never been isolated: magnetic poles always found in pairs „N” and „S”: there are no magnetic monopoles
Magnetic field patterns of bar magnet using small iron filings:

- Magnetic field pattern surrounding a bar magnet
- Magnetic field pattern between opposite poles (N–S) of two bar magnets
- Magnetic field pattern between like poles (N–N) of two bar magnets
Magnetic Field

- If bar magnet suspended from its midpoint and can swing freely in horizontal plane it will rotate until its magnetic north pole points toward Earth’s geographic North Pole (the North geomagnetic pole is actually the south pole of the Earth's magnetic field, and the South geomagnetic pole is the north pole).

- Direction of magnetic field $\mathbf{B}$ at any location is direction in which north pole of compass needle points at that location

- Figure: shows how magnetic field lines of bar magnet can be traced using compass

- Note: magnetic field lines outside magnet point away from north pole and toward south pole
Earth’s Magnetic Field

- South pole Earth’s magnetic field located near north geographic pole
- Magnetic field is horizontal with respect to Earth’s surface near equator
- Field points more and more toward Earth’s surface as you move north
- At north magnetic pole, field points directly down
Earth’s Magnetic Field

- Earth’s magnetic poles move:

- North and south magnetic poles are not on opposite sides on Earth:

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2004</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Magnetic Pole</td>
<td>81.3°N</td>
<td>82.3°N</td>
<td>83.95°N</td>
</tr>
<tr>
<td></td>
<td>110.8°W</td>
<td>113.4°W</td>
<td>120.72°W</td>
</tr>
<tr>
<td>South Magnetic Pole</td>
<td>64.6°S</td>
<td>63.5°S</td>
<td>64.497°S</td>
</tr>
<tr>
<td></td>
<td>138.5°E</td>
<td>138.0°E</td>
<td>137.684°E</td>
</tr>
</tbody>
</table>

- The geomagnetic poles are antipodal points where the axis of a best-fitting dipole intersects the Earth's Surface. This dipole is equivalent to a powerful bar magnet at the center of the Earth, and it is this theoretical dipole that comes closer than any other to accounting for the magnetic field observed at the Earth's surface.
Earth’s Magnetic Field

**Intensity**: The Earth's field ranges between approximately 25,000 and 65,000 nT:

\[25 \text{ -- } 65 \, \mu T\]

\[(25 \text{ -- } 65) \times 10^{-6} T\]

**Inclination**: the angle made with the horizontal by the Earth's magnetic field lines.

**Declination**: the angle on the horizontal plane between magnetic north (the direction the north end of a compass needle points, corresponding to the direction of the Earth's magnetic field lines) and true north (the direction along a meridian towards the geographic North Pole).
Storyline

- Number in runway marker in chapter opening photograph refers to direction of runway relative to magnetic north, divided by 10
- Runway 35 oriented 350°, measured clockwise, from magnetic north
- R means at least two parallel runways in this direction, and this one is on the right
- Other runway marked 35L (left)
- Other possibility: 35C (center)

- At Vancouver International Airport: two runways at orientation of 100° relative to true north
- Because of magnetic declination: not marked as runway 10, but rather as runways 8L and 8R, because oriented 83° from magnetic north
Particle in a Magnetic Field

- Magnetic force proportional to charge \( q \) of particle.
- Magnetic force proportional to speed \( v \) of particle.
- Magnetic force on negative charge directed opposite to force on positive charge moving in same direction.
- Magnetic force proportional to magnitude of magnetic field vector \( \mathbf{B} \).
Particle in a Magnetic Field

\[ \vec{F}_B = q\vec{v} \times \vec{B} \]

The magnetic force is perpendicular to both \( \vec{v} \) and \( \vec{B} \).

The magnetic forces on oppositely charged particles moving at the same velocity in a magnetic field are in opposite directions.
Particle in a Magnetic Field

\[ \vec{F}_B = q \vec{v} \times \vec{B} \]

\[ F_B = |q| v B \sin \theta \]

\[ 1 \text{ T} = \frac{N}{C \cdot \text{m/s}} \]

\[ 1 \text{ T} = \frac{N}{A \cdot \text{m}} \]

\[ 1 \text{ T} = 10^4 \text{ G} \]

**TABLE 28.1** Some Approximate Magnetic Field Magnitudes

<table>
<thead>
<tr>
<th>Source of Field</th>
<th>Field Magnitude (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong superconducting laboratory magnet</td>
<td>30</td>
</tr>
<tr>
<td>Strong conventional laboratory magnet</td>
<td>2</td>
</tr>
<tr>
<td>Medical MRI unit</td>
<td>1.5</td>
</tr>
<tr>
<td>Bar magnet</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Surface of the Sun</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Surface of the Earth</td>
<td>$5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Inside human brain (due to nerve impulses)</td>
<td>$10^{-13}$</td>
</tr>
</tbody>
</table>
Comparison of Electric and Magnetic Force

<table>
<thead>
<tr>
<th>Electric force vector along direction of electric field</th>
<th>Magnetic force vector perpendicular to magnetic field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric force acts on charged particle regardless of whether particle is moving</td>
<td>Magnetic force acts on charged particle only when particle is in motion</td>
</tr>
<tr>
<td>Electric force does work in displacing a charged particle</td>
<td>Magnetic force associated with steady magnetic field does no work when a particle is displaced → force perpendicular to displacement of its point of application</td>
</tr>
</tbody>
</table>
Particle in a Magnetic Field

\[ \vec{F}_B = q\vec{v} \times \vec{B} \]

Examples:

- An ion moves in a circular path in the magnetic field of a mass spectrometer
- A coil in a motor rotates in response to the magnetic field in the motor
- A magnetic field is used to separate particles emitted by radioactive sources
- In a bubble chamber, particles created in collisions follow curved paths in a magnetic field, allowing the particles to be identified
Motion of a Charged Particle in a Uniform Magnetic Field with \( \mathbf{v} \perp \mathbf{B} \)

\[
\sum F = F_B = ma
\]

\[
F_B = qvB = \frac{mv^2}{r}
\]

\[
r = \frac{mv}{qB}
\]

\[
\omega = \frac{v}{r} = \frac{qB}{m}
\]

\[
T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}
\]

The magnetic force \( \mathbf{F}_B \) acting on the charge is always directed toward the center of the circle.
Motion of a Charged Particle in a Uniform Magnetic Field

1) $\mathbf{v}$ is perpendicular to $\mathbf{B}$:

$$ r = \frac{mv}{qB} \quad \omega = \frac{v}{r} = \frac{qB}{m} $$

$$ T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB} $$

Path is circle

2) $\mathbf{v}$ is not perpendicular to $\mathbf{B}$:

$$ v \equiv v_{\perp B} = \sqrt{v_y^2 + v_z^2} = \text{const} $$

$$ v_{\parallel B} = \text{const} $$

Path is helix
Van Allen Radiation Belts

- Van Allen radiation belts consist of charged particles (mostly electrons and protons) surrounding Earth in doughnut-shaped regions.
- Particles trapped by Earth’s nonuniform magnetic field:
  - Spiral around field lines from pole to pole.
  - Covering distance in only a few seconds.
- These particles originate mainly from the Sun:
  - Some come from stars and other heavenly objects.
  - Called cosmic rays.
- Most cosmic rays deflected by the Earth’s magnetic field and never reach atmosphere:
  - Some particles become trapped and make up the Van Allen belts.
Auroras

- When particles located over poles:
  - Sometimes collide with atoms in atmosphere
  - Atoms emit visible light
- Such collisions are origin of aurora borealis (northern lights) in northern hemisphere and aurora australis in southern hemisphere
Auroras

- Auroras usually confined to polar regions because Van Allen belts are nearest Earth’s surface there
  - Occasionally solar activity causes larger numbers of charged particles to enter belts and significantly distort normal magnetic field lines associated with Earth
    - Auroras sometimes seen at lower latitudes
Velocity Selector

\[ \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \]

When magnitudes of two fields chosen so that:

\[ qE = qvB, \]

forces cancel.

Only particles having speed

\[ v = \frac{E}{B} \]

pass undeflected through mutually perpendicular electric and magnetic fields.
**The Mass Spectrometer**

**Mass spectrometer:** separates ions according to mass-to-charge ratio

Beam of ions passes through velocity selector and enters second uniform magnetic field $B_0$ (has same direction as magnetic field in selector). Ions in second magnetic field move in semicircle of radius $r$ (described by particle in uniform circular motion model)

$$r = \frac{mv}{qB}$$

before striking detector array. Positively charged ions are deflected left, negatively – right.

The mass-to-charge ratio is determined by measuring radius of curvature and knowing electric and magnetic field magnitudes:

$$\frac{m}{q} = \frac{rB_0B}{E}$$

In practice: usually mass spectrometer measure masses of various isotopes of a given ion.
Measuring $e/m_e$ (J. J. Thomson 1897)

Electrons are accelerated from the cathode, pass through two slits, and are deflected by both an electric field (formed by the charged deflection plates) and a magnetic field (directed perpendicular to the electric field). The beam of electrons then strikes a fluorescent screen.

$$\frac{e}{m} = C \cdot \frac{E}{B^2} \gamma$$

J. J. Thomson and Frank Baldwin Jewett
Cyclotron: device that accelerates charged particles to very high speeds.

\[ T = \frac{2\pi m}{qB} \]

\[ v = \frac{qBr}{m} \Rightarrow K = \frac{1}{2}mv^2 = \frac{q^2B^2r^2}{2m} \]
Magnetic Force Acting on a Current-Carrying Conductor

When there is no current in the wire, the wire remains vertical.

When the current is upward, the wire deflects to the left.

When the current is downward, the wire deflects to the right.

\[ q\vec{v}_e \times \vec{B} = -e\vec{v}_e \times \vec{B} \]
Magnetic Force Acting on a Current-Carrying Conductor

\[ \vec{F}_B = q \vec{v}_d \times \vec{B} \]

charges in segment: \( N = nV = nAL \)  
\((n = \text{number of mobile charge carriers per unit volume})\)

\[ \vec{F}_B = \left( q \vec{v}_d \times \vec{B} \right) nAL \]

\[ I = nq\nu_d A \]

\[ \vec{F}_B = I\vec{L} \times \vec{B} \]

- \( \vec{L} \) – vector:
  - points in direction of current \( I \)
  - has magnitude = length \( L \) of segment

The average magnetic force exerted on a charge moving in the wire is \( q\vec{v}_d \times \vec{B} \).
Expression \( \vec{F}_B = I \vec{L} \times \vec{B} \)

applies only to straight segment of wire in uniform magnetic field.

Magnetic force exerted on small segment of vector length \( ds \) in presence of field \( \vec{B} \) is:

\[
d\vec{F}_B = Id\vec{s} \times \vec{B}
\]

Total force \( \vec{F}_B \) acting on wire \( d\vec{s} \):

\[
\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B}
\]

– integral over total length of wire
(a and b represent endpoints of wire)
Rectangular loop carrying current $I$ in presence of uniform magnetic field $\mathbf{B}$ directed parallel to plane of loop.

No magnetic forces act on sides 1 and 3: these wires are parallel to field, $\mathbf{L} \times \mathbf{B} = \mathbf{0}$ for these sides.

Magnetic forces act on sides 2 and 4: these wires are perpendicular to field, magnitude of force:

$$F_2 = F_4 = IaB \sin(90^\circ) = IaB$$

These two forces produce torque ($\tau = \mathbf{r} \times \mathbf{F}$) about $O$

$$\tau_{\text{max}} = -F_2 \frac{b}{2} - F_4 \frac{b}{2}$$

$$= -(IaB) \frac{b}{2} - (IaB) \frac{b}{2}$$

$$= -IabB = -BIS$$

that rotates loop clockwise.
Torque on a Current Loop in a Uniform Magnetic Field

\[ \tau = -F_2 \frac{b}{2} \sin \theta - F_4 \frac{b}{2} \sin \theta \]

\[ = -IaB \left( \frac{b}{2} \sin \theta \right) - IaB \left( \frac{b}{2} \sin \theta \right) = -IabB \sin \theta \]

\[ = -BIS \sin \theta \]
An electric motor uses electric current to create rotational motion.

When an electric current flows along a piece of wire in a magnetic field, the wire moves. In a simple motor, the wire is in the form of a coil placed between the poles of a magnet.

The ends of the coil are attached to a pair of split rings called a commutator. Current is fed to the commutator through a pair of carbon brushes. When the wires move in the magnetic field, the coil rotates.
A galvanometer detects and measures the current flowing through a wire using the rotation of the wire loop in a uniform magnetic field.

The current loop (coil) in a magnetic field rotates in a manner that depends on the magnitude and direction of the current flowing through the loop.

A galvanometer can be used as an ammeter and can be a sensitive detector of small currents (nA).
The Hall Effect

When current-carrying conductor is placed in magnetic field, potential difference is generated between two points lying along direction perpendicular to both current and magnetic field.

- Flat conductor carrying current $I$ in $x$ direction
  - Uniform magnetic field $\mathbf{B}$ applied in $y$ direction
- If charge carriers are electrons moving in negative $x$ direction with drift velocity $v_d$
  - Experience upward magnetic force $\mathbf{F}_B = qv_d \times \mathbf{B}$
    - Deflected upward
    - Accumulate at upper edge of flat conductor
    - Excess of positive charge at lower edge (bottom figure (a))

- Accumulation of charge at edges establishes electric field in conductor
  - Increases until electric force on carriers remaining in bulk of conductor balances magnetic force acting on carriers

$$ qv_d B = qE_H \implies E_H = v_d B $$
The Hall Effect

- Moving electrons no longer deflected upward
- Sensitive voltmeter connected across sample →
  - Measure potential difference (**Hall voltage** $\Delta V_H$) generated across conductor

$$\Delta V_H = E_H d = v_d B d$$

$$\Delta V_H = \frac{IBd}{nqA}$$

$$\Delta V_H = \frac{IB}{nqt}$$

$$= \frac{R_H}{t} IB$$

$\nu_d = \frac{I}{nqA}$

$A = td$

A **Hall effect sensor** can measure a wide range of magnetic fields or may operate as an electronic switch.
A rectangular copper strip 1.5 cm wide and 0.10 cm thick carries a current of 5.0 A. Find the Hall voltage for a 1.2-T magnetic field applied in a direction perpendicular to the strip.
Example 28.6: The Hall Effect for Copper

\[ \Delta V_H = \frac{IB}{nqt} \]

\[ n = \frac{N_A}{V} = \frac{N_A \rho}{M} \]

- \( M \) – molar mass
- \( \rho \) – density
- \( N_A \) – Avogadro constant

\[ n = \frac{0.0635 \text{ kg/mol}}{(6.02 \times 10^{23} \text{ mol}^{-1})(8920 \text{ kg/m}^3)} = 1.18 \times 10^{29} \text{ m}^{-3} \approx 1 \times 10^{23} \text{ cm}^{-3} \]

\[ \Delta V_H = \frac{(5.0 \text{ A})(1.2 \text{ T})}{(1.18 \times 10^{29} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(0.001 \text{ m})} = 0.44 \mu \text{V} \]
Example 28.6: The Hall Effect for Semiconductor

What if the strip has the same dimensions but is made of a semiconductor? Will the Hall voltage be smaller or larger?

\[ \Delta V_H = \frac{IB}{nqt} \]

\[ \Delta V_H = \frac{(0.1 \text{ mA})(1.2 \text{ T})}{(1.0 \times 10^{20} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(0.001 \text{ m})} = 7.5 \text{ mV} \]